

# Aspiration-induced reconnection in spatial public goods game

HAI-FENG ZHANG<sup>1,2</sup> (a), RUN-RAN LIU<sup>2</sup>, ZHEN WANG<sup>3</sup> (b), HAN-XIN YANG<sup>2</sup> and BING-HONG WANG<sup>2</sup> (c)

<sup>1</sup> School of Mathematical Science, Anhui University, Hefei 230039, P. R. China

<sup>2</sup> Department of Modern Physics, University of Science and Technology of China, Hefei, 230026, China

<sup>3</sup> School of Physics, Nankai University, Tianjin, 300071, P. R. China

PACS 87.23.Ge – Dynamics of social systems

PACS 02.50.Le – Decision theory and game theory

PACS 87.23.Cc – Population dynamics and ecological pattern formation

**Abstract.** - In this Letter, we introduce an aspiration-induced reconnection mechanism into the spatial public goods game. A player will reconnect to a randomly chosen player if its payoff acquired from the group centered on the neighbor does not exceed the aspiration level. We find that an intermediate aspiration level can best promote cooperation. This optimal phenomenon can be explained by a negative feedback effect, namely, a moderate level of reconnection induced by the intermediate aspiration level induces can change the downfall of cooperators, and then facilitate the fast spreading of cooperation. While insufficient reconnection and excessive reconnection induced by low and high aspiration levels respectively are not conducive to such an effect. Moreover, we find that the intermediate aspiration level can lead to the heterogeneous distribution of degree, which will be beneficial to the evolution of cooperation.

**Introduction.** – Cooperation is ubiquitous in biological and social systems, yet, understanding the emergence and maintenance of cooperative behaviors is still a major challenge [1]. In order to resolve this puzzle, evolutionary game theory provides a fruitful framework to address the evolution of cooperation among unrelated individuals [2, 3]. For example, the prisoner’s dilemma game [4, 5], the snowdrift game [4, 6] and the stag-hunt game [7, 8] have been intensively studied as the paradigms to investigate cooperative behaviors through pairwise interactions. However, some social dilemmas involve larger groups of interactional individuals, as can be observed by these phenomena: resource distribution and redistribution, predator inspection behavior, alarm calls, and group defense, health insurance, public transportation and environmental issues. In such cases, public goods game rather than the game of pairwise interactions seems suited to provide reasonable explanation for the facilitation of cooperation [9–19].

In the original public goods game consisting of  $N$  players, each player can decide whether to contribute an amount  $c$  to the common pool (cooperation) or not (defection). Whereafter, the overall contributions are multiplied

by a multiplication factor  $r$ , and then are redistributed equally among all the players, irrespective of their initial contributions to the common pool [17]. It’s obvious that selfish players are enforced upon to select defection, which can take advantage of the public goods. In the traditional game theory, players are perfectly rational, thus defection becomes the dominant strategy leading to the deterioration of cooperation, which is known as the Tragedy of the Commons [20, 21].

Over the past decades, a number of theoretical and experimental evidences have been proposed and investigated to understand the emergence of cooperation. Remarkable mechanisms include kin selection [22], direct and indirect reciprocity [23, 24], punishment and reward [25, 26], heterogeneous activity [16], group selection [27], voluntary participation [9, 10, 13], spatial effects [10, 28, 29], image score effect [30, 31], success-driven migration [32], to name but a few. Importantly, apart from the considerable attention paid to the facilitation mechanisms alone, the coevolutionary game also attracts great interest [33–45]. Since it not only reflects the evolving of strategies over time, but also characterizes the adaptive development of the network topologies or the update rules (for a further review see [33]). For instance, in [34–36] the rewiring of existing links was recognized as very beneficial to the evolution of cooperation, the growth of a network had a positive

(a) haifeng3@mail.ustc.edu.cn

(b) zhenwang@mail.nankai.edu.cn

(c) bhwang@ustc.edu.cn

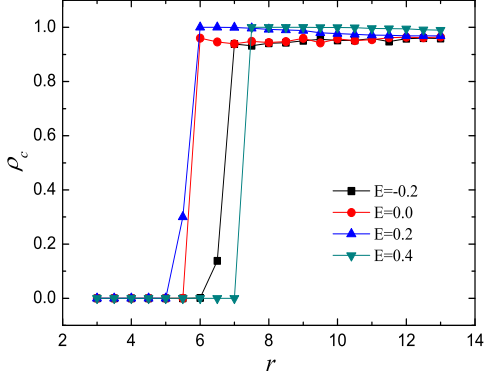


Fig. 1: ((Color online) Fraction of cooperators  $\rho_c$  in dependence on the multiplication factor  $r$  for different values of aspiration level  $E$ . Note that intermediate value of  $E$  can sustain cooperation better than lower or larger case.

impact on the evolution of cooperation in [41–43], and cooperation could also be promoted when the coevolution of strategies and update rules were considered [44, 45]. Take some multinational corporations as examples, these corporations often extend their business to different countries or regions to pursue their maximal profit. However, once the profit gained from a country or region is undesirable, they will withdraw the investment partially or entirely, and then transfer it to other countries or regions. Inspired by these actual phenomena and the plentiful achievements of coevolutionary, in the Letter, we propose an aspiration-induced reconnection mechanism to study the emergence of cooperation in the public goods game. In the game, if player's payoff obtained from the group centered on one of its neighbors does not exceed the aspiration level, the player will cut the link with the neighbor and rewire the link to one randomly selected player. Interestingly, we find that such a simple but meaningful approach can promote the level of cooperation best under a moderate aspiration level, while it is not conducive to facilitate cooperation for too low or too high aspiration levels. We give an interpretation to these observed phenomena by inspecting the process of evolution and investigating the degree distribution of the evolved network. Moreover, we examine the universality of such a mechanism through the variation of game model.

The remainder of this paper is organized as follows. In Sec. II, we will first describe the model of considered evolutionary game. Then we present the main results and discussions in Sec. III. Finally, we will summarize the conclusion in Sec. IV.

**Evolutionary game.** — We consider the public goods game with players located on the nodes of the spatial network. According to Ref. [18], each individual  $i$  participates in interactions in  $k_i + 1$  neighborhoods that center about  $i$

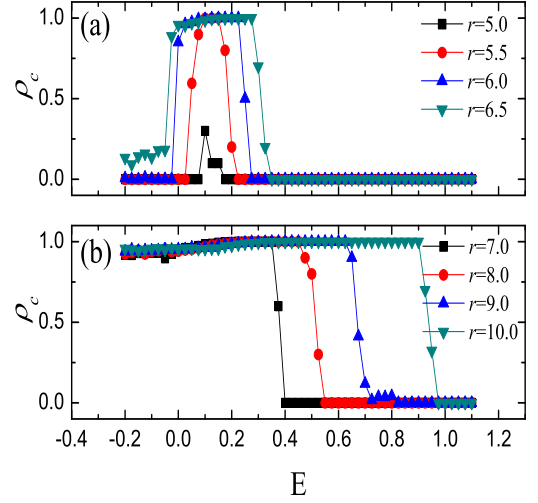


Fig. 2: (Color online) Fraction of cooperators  $\rho_c$  in dependence on aspiration level  $E$  for different values of multiplication factors  $r$ . Note that for  $r \leq 6.5$  (a), it can be obviously observed that there exists an intermediate aspiration level  $E$  for which the evolution of cooperation is optimal; while for  $r \geq 7$  (b), this peak is not distinct as the above case because of the fact that large  $r$  can promote cooperation better, even though the invariance of network.

and its  $k_i$  neighbors, where each neighborhood contains a central node and all nodes that are directly connected to it. Each cooperator contributes a total cost  $c = 1$  shared equally among all the neighborhoods that it engages. The strategy is  $s_x = 1$  for a  $C$  player and  $s_x = 0$  otherwise. The payoff of the individual  $x$  with strategy  $s_x$  associated with the neighborhood centered at an individual  $y$  is given by

$$p_{x,y} = \frac{r}{k_y + 1} \sum_{i=0}^{k_y} \frac{s_i}{k_i + 1} - \frac{s_x}{k_x + 1}, \quad (1)$$

where  $i = 0$  represents  $y$ ,  $s_i$  is the strategy of the neighbors  $i$  of  $y$ , and  $k_i$  is its degree. The total payoff of player  $x$  is

$$P_x = \sum_{y \in \Lambda_x} p_{x,y}, \quad (2)$$

where  $\Lambda_x$  is the neighborhood of  $x$  and itself [17].

As the interaction network, we use a Newman-Watts small world network, where a number of long-range links  $N_{add}$  are randomly added to the two-dimensional lattice with periodic boundary conditions [46]. Initially, each player is designed to be either a cooperator or defector with equal probability. Players asynchronously update their strategies in a random sequential order. Before updating strategy, the randomly selected player  $x$  evaluates the payoff  $p_{x,y}$  from the neighborhood centered on  $y$  according to Eq. (1). If the payoff  $p_{x,y}$  cannot satisfy the

aspiration level  $E$  ( $E$  is uniform for all the players), player  $x$  will remove the link with neighbor  $y$ , and then create a new link to a randomly chosen non-neighbor node from network (i.e., multiple links are prohibited.) In our model, we assume that the local interactions with four nearest neighbor nodes (the von Neumann neighborhood) do not engage in the reconnection behaviors. It is because that these local interactions are determined by the spatial neighborhoods, and they are reasonably assumed to be fixed during the whole process [37]. After rewiring, player  $x$  collects its total payoff according to Eq. (2), then all the neighbors of player  $x$  acquire their payoffs by means of the same way as player  $x$ .

Lastly, the player  $x$  will randomly select one of its neighbors  $z$  and adopts  $z$ 's strategy with a probability  $W(s_z \rightarrow s_x)$  depending on the payoff difference. Namely

$$W(s_z \rightarrow s_x) = \frac{1}{1 + \exp[(P_x - P_z)/K]}, \quad (3)$$

where  $K$  denotes the amplitude of noise or its inverse ( $1/K$ ), the so-called intensity of selection [47]. In the limit  $K \rightarrow 0$ , the strategy of neighbor  $y$  is always adopted provided that  $P_y > P_x$ . While in the limit  $K \rightarrow \infty$  all information is lost, that is, player  $x$  switches to the strategy of neighbor  $y$  by tossing a coin. Furthermore, it should be noted that this update rule is the sequential version of the Fermi rule, at variance with some other works in which updates are made in parallel [17, 19]. Since the effect of  $K$  on the evolution of cooperation has been studied in detail in Refs. [19, 48, 49], we simply set the value of  $K$  to be  $K = 0.1$ .

**Results and discussions.** — Results of Monte Carlo simulations presented below are obtained on a Newman-Watts small world network hosting  $N = 2500$  players with average degree  $\langle k \rangle = 8$ , namely, the number of long-range links is  $N_{add} = 2N$  (By extensive simulations, we find our results are robust to the network sizes and the average degrees.). On average, in each Monte Carlo step (MCS), all players have the chances of rewiring and updating their strategies. The key quantity for characterizing the cooperative behavior is the fraction of cooperators  $\rho_c$ , which is gained by averaging 2000 full steps after a transient time of 10000 generations in our work. Moreover, since the process of rewiring may yield heterogeneous distribution of links, which will seriously affects the accuracy of simulations. Therefore, the final results are averaged over 40 independent runs for each parameter value.

Figure 1 shows the fraction of cooperators  $\rho_c$  as a function of the multiplication factor  $r$  for different aspiration levels  $E$ . One can see that, for  $E = 0.0$  or  $E = 0.2$ , the cooperative phenomenon can be flourished even when  $r \leq 5.5$  (especially, for the case of  $E = 0.2$ ,  $\rho = 1$  when  $r = 5.5$ ), yet for  $E = -0.2$  or  $E = 0.4$ , cooperative phenomenon appears only for  $r \geq 6$ . These results suggest that there may exist an intermediate aspiration level  $E$ ,

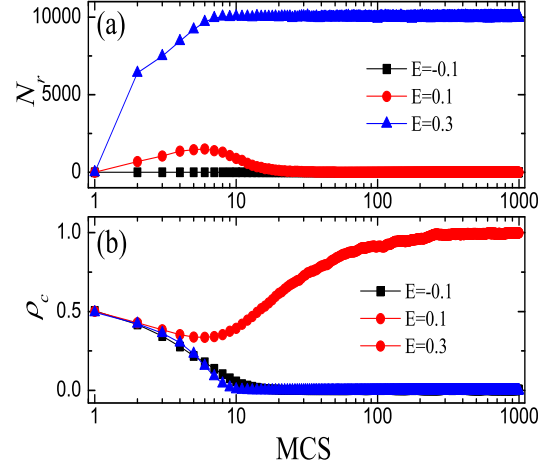


Fig. 3: (Color online) Time courses depicting the number of reconnections  $N_r$  and the fraction of cooperators  $\rho_c$  for different values of  $E$ . Note in (a), the number of reconnections will increase at first for intermediate and large aspiration levels, but develops into different patterns finally. While for small aspiration level, the number of reconnections will keep constant. For the evolution of cooperation in (b), the intermediate aspiration level will go through a negative feedback effect to promoting cooperation, which corresponds to a peak of reconnections, while small and large aspiration levels can not avoid the destiny of cooperation dying out. All the results are obtained for  $r = 6.0$ .

which induces the optimal cooperation. To examine the effect of  $E$  on the evolution of cooperation more precisely, we present  $\rho_c$  in dependence on the aspiration level  $E$  for different values of  $r$  in Fig. 2. As shown in Fig. 2(a), the impact of small  $E$  on cooperation remains marginal, and thus most cooperators could not resist the exploitation against defectors or only a small fraction of cooperators could survive. However, as aspiration level  $E$  reaches an intermediate value, a remarkable increase of cooperation can be observed. By further increasing the aspiration level, the facilitation effect is deteriorated again. These results favor that the intermediate aspiration level can warrant an optimal promotion of cooperation, which is analogous to the so-called coherence resonance [50, 51]. For large  $r$  (i.e.,  $r \geq 7$ ), as shown in Fig. 2(b), the non-monotonic phenomenon is not so distinct as the case of small  $r$ . This is actually what one would expect, because, compared with small  $r$  where the sum of cooperators is very limited or zero, large  $r$  can make sure of generating more rewards for the cooperation, even though the network does not evolve.

In order to understand the nontrivial effect of aspiration level on the fraction of cooperators, we study the time courses of reconnection number  $N_r$  and the fraction of cooperators  $\rho_c$  for different values of  $E$  in Fig. 3. For

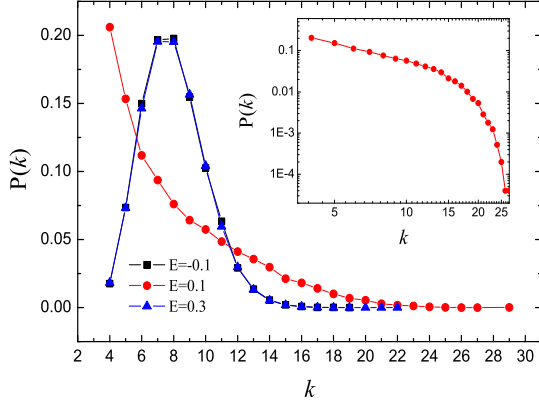


Fig. 4: (Color online) Degree distributions  $P(k)$  of networks for different aspiration levels  $E$  in the final steady states. Note that small and large aspiration levels will not alter the initial norm distribution with  $\langle k \rangle = 8$ , while intermediate aspiration level induces an explicit heterogeneous distribution that is more beneficial for the evolution of cooperation (see also the inset for  $E = 0.1$ ). All the results are obtained as averages over 50 independent realizations for  $r = 6.0$ .

$E = -0.1$ , Figure 3(a) illustrates that due to the fact that individual payoffs are always higher than aspiration level, the initial topology structure of interaction network does not change over time, namely, the number of reconnections always equals zero. At this time, cooperators on the initial network cannot resist the exploitation from defectors, which results in the extinction of cooperation (see Fig. 3(b)). Whereas for large aspiration level ( $E = 0.3$ ), most individuals' payoffs are lower than the aspiration level. As a result, reconnection behavior occurs frequently and individuals will randomly connect to others as time evolves. In such case, the cooperative clusters are fragile and are easy to be destroyed by the invasion of the defectors. So large aspiration level also induces the extinction of cooperation. More significantly, we observe that the immediate aspiration level ( $E = 0.1$ ) can induce a peak of reconnections which yet corresponds to a downside for the evolution of cooperation. In the most early stages of process, defection yields higher individual benefit, and the outlook for cooperators is gloomy. With the game forward, a few players' payoffs could not exceed the aspiration level, and thus a small fraction of reconnections will appear. All the reconnections will effectively alter the evolution tide, namely, the downfall of cooperators will transform into the fast spreading of cooperators. Because, in such case, only choosing cooperation adequately can exceed the aspiration level. When cooperation becomes the dominant strategy, individual payoffs will uniformly exceed the aspiration level, which makes the possibility of reconnection vanish, namely, the number of reconnections will become zero over again. Consequently, an intermediate aspiration

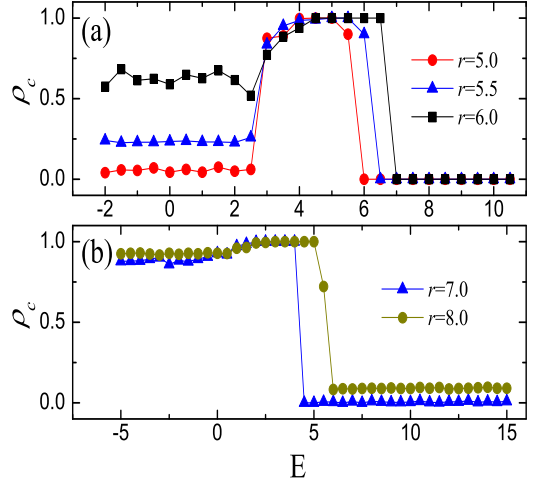


Fig. 5: (Color online) Fraction of cooperators  $\rho_c$  in dependence on aspiration level  $E$  for different values of multiplication factor  $r$  in the game where every cooperator separately contributes a cost  $c = 1$  to each neighborhood that it engages. Note that the optimal promotion phenomena of cooperation are similar to those presented in Fig. 2.

level can result in a peak of reconnections which promotes the level of cooperation by means of a negative feedback effect. While for low or high aspiration levels, insufficient or excessive reconnections could not provide advantageous conditions for such a feedback effect.

Subsequently, it remains of interest to examine the degree distributions of the evolved networks for different aspiration levels. It has been known that the heterogeneity of host network plays an important role in the substantial promotion of cooperation. If the hub node takes the cooperation (defection) strategy, its strategy will become an example to be imitated by its neighborhood, which increases (decreases) cooperators' (defectors') payoffs and results in the great promotion of cooperation [4, 17, 18]. Hence, what we would expect is to present a highly heterogeneous distribution as well. Figure 4 demonstrates clearly the degree distributions in the final steady states. According to the time courses of  $N_r$  in Fig. 3(a), we observe that the network does not evolve with time for  $E = -0.1$ , so the degree distribution obeys norm distribution with  $\langle k \rangle = 8$  (see Fig. 4). While for  $E = 0.3$ , long-range links which are generated randomly and excessively make the degree distribution confirm to the norm distribution likewise. Interestingly, for  $E = 0.1$ , the degree distribution deviates from the norm distribution and the heterogeneity of network is formed (see also the inset of Fig. 4 by using a log-log representation), which is consistent with our expectation. The highly heterogeneous distribution induced by the intermediate aspiration level is crucial for the op-



timal promotion phenomenon of cooperation presented in Fig. 2. Therefore, the promotion of cooperation partly attributes to the potential heterogeneous states within the network.

In the above evolutionary game, we assume that each cooperator contributes the same total cost  $c = 1$ , which is then shared equally among all the neighborhoods that it engages in, namely, the individual contribution is independent of the number of its social ties. Whereas, in the opposite limit considered in Ref. [18], every cooperator separately contributes a cost  $c = 1$  to each neighborhood that it engages. In this case, the total contribution of each cooperator  $x$  is equal to  $k_x + 1$ . Similarly as in Eq. (1), the payoff of player  $x$  (with strategy  $s_x$ ) obtained from the neighborhood centered on the player  $y$  is given by

$$p_{x,y} = \frac{r}{k_y + 1} \sum_{i=0}^{k_y} s_i - s_x. \quad (4)$$

Interestingly, as shown in Fig. 5, there also exists an intermediate aspiration level, leading to the highest cooperation level when  $r$  is fixed. Thus, the aspiration-induced reconnection mechanism is robust for promoting cooperation, regardless of the total contribution of cooperators.

**Summary.** — In summary, we have studied the effect of the aspiration-induced reconnection on cooperation in spatial public goods game. In the game, if player's payoff acquired from the group centered on the neighbor does not exceed aspiration level, it will remove the link with the neighbor and reconnect it to a randomly selected player. Through scientific simulations, we have shown that, irrespective of the total contribution of each cooperator, there exists an intermediate aspiration level results in the optimal cooperation. This optimal phenomenon can be explained by means of a negative feedback effect. In the early stages of evolutionary process, though cooperators are decimated by defectors, the peak of reconnection induced by the intermediate aspiration level will change the downfall of cooperation and facilitate the fast spreading of cooperation. While for too low or too high aspiration levels, insufficient or excessive reconnections does not provide any possibility for the emergence of such a feedback effect. Moreover, we have analyzed the degree distributions of the network. Of particular interest is that the heterogeneous degree distribution induced by intermediate aspiration level, warrants a potent promotion of cooperation. Since the phenomena related with aspiration-induced reconnection are abundant in our society, we hope that our results can offer a new insight into understanding these phenomena.

\* \* \*

This work is funded by the National Natural Science Foundation of China (Grant No. 10975126, 10635040, 11047012) and the Specialized Research Fund

for the Doctoral Program of Higher Education of China (Grant No. 20060358065). HZ is funded by the 211 Project of Anhui University (2009QN003A, KJTD002B), and the National Natural Science Foundation of China (Grant No. 11005001). ZW is funded by the Center for Asia Studies of Nankai University (Grant No. 2010-5) and the National Natural Science Foundation of China (Grant No. 10672081 and Grant No. 60904063).

## REFERENCES

- [1] Colman A. M., *Game Theory and Its Applications in the Social and Biological Sciences* (Butterworth-Heinemann, Oxford.) 1995.
- [2] Smith J. M., *Evolution and the Theory of Games* (Cambridge University Press, Cambridge, England.) 1982.
- [3] Gintis H., *Game Theory Evolving* (Princeton University Press, Princeton, NJ.) 2000.
- [4] Santos F. C. and Pacheco J. M., *Phys. Rev. Lett.*, **95** (2005) 098104.
- [5] Santos F. C., Lenaerts T. and Pacheco J. M., *Phys. Rev. Lett.*, **102** (2009) 058105; Perc M. and Wang Z., *PLoS ONE*, **5** (2010) e15117; Perc M., *EPL*, **75** (2006) 841; Szolnoki A. and Szabó G., *EPL*, **77** (2007) 30004; Kim B. J., Trusina A., Holme P., Minnhagen P., Chung J. S. and Choi M. Y., *Phys. Rev. E*, **66** (2002) 021907; Wang Z. and Perc M., *Phys. Rev. E*, **82** (2010) 021115; Zhang H.-F., Small M., Yang H.-X. and Wang B.-H., *Physica A*, **389** (2010) 4734; Gómez-Gardeñes J., Campillo M., Floría L. M. and Moreno Y., *Phys. Rev. Lett.*, **98** (2007) 108103; Wu Z.-X. and Holme P., *Phys. Rev. E*, **80** (2009) 026108.
- [6] Hauert C. and Doebeli M., *Nature*, **428** (2004) 643; Wang W.-X., Ren J., Chen G.-R. and Wang B.-H., *Phys. Rev. E*, **74** (2006) 056113; Wang Z., Du W.-B., Cao X.-B. and Zhang L.-Z., *Physica A*, **390** (2011) 1234; Du W.-B., Cao X.-B., Hu M.-B. and Wang W.-X., *EPL*, **87** (2009) 60004.
- [7] Roca C. P., Cuesta J. A. and Sanchez A., *Phys. Rev. Lett.*, **97** (2006) 158701.
- [8] Szolnoki A., Vukov J. and Szabó G., *Phys. Rev. E*, **80** (2009) 056112.
- [9] Hauert C., De Monte S., Hofbauer J. and Sigmund K., *Science*, **296** (2002) 1129.
- [10] Szabó G. and Hauert C., *Phys. Rev. Lett.*, **89** (2002) 118101.
- [11] Hauert C., De Monte S., Hofbauer J. and Sigmund K., *J. Theor. Biol.* **218** (2002) 187.
- [12] Rong Z.-H. and Wu Z.-X. *EPL*, **87** (2009) 30001.
- [13] Semmann D., Krambeck H. J. and Milinski M., *Nature*, **425** (2003) 390.
- [14] Brandt H., Hauert C. and Sigmund K., *Proc. Natl. Acad. Sci.*, **103** (2006) 495.
- [15] Xu Z.-J., Wang Z., Song H.-P. and Zhang L.-Z., *EPL*, **90** (2010) 20001.
- [16] Guan J.-Y., Wu Z.-X. and Wang Y.-H., *Phys. Rev. E*, **76** (2007) 056101.
- [17] Yang H.-X., Wang W.-X., Wu Z.-X., Lai Y.-C. and Wang B.-H., *Phys. Rev. E*, **79** (2009) 056107.
- [18] Santos F. C., Santos M. D. and Pacheco J. M., *Nature*, **454** (2008) 213.

- [19] Szolnoki A., Perc M. and Szabó G., *Phys. Rev. E*, **80** (2009) 056109.
- [20] Hardin G., *Science*, **162**, (1968) 1243.
- [21] Hardin G., *Science*, **280**, (1998) 682.
- [22] Hamilton W. D., *J. Theor. Biol.*, **7** (1964) 1.
- [23] Trivers R. L., *Q. Rev. Biol.*, **46** (1971) 35.
- [24] Nowak M. A. and Sigmund K., *Nature*, **393** (1998) 573.
- [25] Fehr E. and Cächter S., *Nature*, **415** (2002) 137.
- [26] Rockenbach B. and Milinski M., *Nature*, **444**, (2006) 718.
- [27] Traulsen A. and Nowak M. A., *Proc. Natl. Acad. Sci. USA*, **103** (2006) 10952.
- [28] Nowak M.A. and May R. M., *Int. J. Bifurcat. Chaos*, **3** (1993) 35.
- [29] Nowak M. A., *Science*, **314** (2006) 1560.
- [30] Brandt H. and Sigmund K., *Proc. Natl. Acad. Sci. U.S.A.*, **102** (2005) 2666.
- [31] Wedekind C. and Milinski M., *Science*, **288** (2000) 850.
- [32] Helbing D. and Yu W., *Proc. Natl. Acad. Sci. U.S.A.*, **106** (2009) 3680; Yang H.-X., Wu Z.-X. and Wang B.-H., *Phys. Rev. E*, **81** (2010) 065101(R).
- [33] Perc M. and Szolnoki A., *BioSystems*, **99** (2010) 109.
- [34] Zimmermann M.G. and Eguíluz V., *Phys. Rev. E*, **72** (2005) 056118.
- [35] Zimmermann M.G., Eguíluz V. and Miguel M.S., *Phys. Rev. E*, **69** (2004) 065102(R).
- [36] Ebel H. and Bornholdt S., *Phys. Rev. E*, **66** (2002) 056118.
- [37] Szolnoki A. and Perc M., *New J. Phys.*, **10** (2008) 043036.
- [38] Li W., Zhang X.-M. and Hu G., *Phys. Rev. E*, **76** (2007) 045102(R).
- [39] Santos F. C., Pacheco J. M. and Lenaerts T., *PLOS Comput. Biol.*, **2** (2006) 10; Pacheco J. M., Traulsen A. and Nowak M. A., *Phys. Rev. Lett.*, **97** (2006) 258103.
- [40] Szolnoki A., Perc M. and Danku Z., *EPL*, **84** (2008) 50007.
- [41] Poncela J., Gómez-Gardeñes J., Floria L. M., Moreno Y. and Sanchez A., *EPL*, **88** (2009) 38003.
- [42] Poncela J., Gómez-Gardeñes J., Moreno Y. and Traulsen A., *New J. Phys.*, **11** (2009) 083031.
- [43] Poncela J., Gómez-Gardeñes J., Floria L. M., Sanchez A. and Moreno Y., *PLoS ONE*, **3** (2008) e2449.
- [44] Moyano L. G. and Sanchez A., *J. Theor. Bio.*, **259** (2009) 84.
- [45] Cardillo A., Gómez-Gardeñes J., Vilone D. and Sanchez A., *New J. Phys.*, **9** (2010) 184.
- [46] Newman M. E. J. and Watts D. J., *Phys. Rev. E*, **60** (1999) 7332.
- [47] Szabó G. and Tóke C., *Phys. Rev. E*, **58** (1998) 69.
- [48] Szabó G. and Vukov J., *Phys. Rev. E*, **69** (2004) 036107.
- [49] Szabó G., Vukov J. and Szolnoki A., *Phys. Rev. E*, **72** (2005) 047107.
- [50] Perc M., *New J. Phys.*, **8** (2006) 22.
- [51] Traulsen A., Röhl T. and Schuster H. G., *Phys. Rev. Lett.*, **93** (2004) 028701.